Indeed, the two theorems, that just given and the previous one,

$$1 - {}_{34\ldots m}r_{12}^{2} = \frac{1 - R^{2}_{1\cdot 234\ldots m}}{1 - R^{2}_{1\cdot 3\cdot 4\ldots m}},$$

are, verbally expressed, identities, the latter having relation to standard deviations measured from *planes* in higher dimensioned space, *i.e.* to multiple "linear" regression—and the former to standard deviations measured from curved *surfaces* in higher dimensioned space, *i.e.* to multiple "skew" regression. The one theorem passes into the other as the skew regression surfaces become planes.

Unfortunately while the rule for finding $H_{1\cdot 23\ldots m}$ is quite simple, the arithmetic is very laborious. The next step in advance must be such a study of skew regression surfaces that we shall learn how to express the multiple correlation ratio in terms of total correlation ratios as we know how to express the multiple correlation coefficient in terms of total correlation coefficients. The first step in this direction has recently been taken by Isserlis in the memoir cited above.

On a Spectrum Associated with Carbon, in Relation to the Wolf-Rayet Stars.

By Thomas R. Merton, B.Sc. (Oxon.), Lecturer in Spectroscopy at University of London, King's College.

(Communicated by A. Fowler, F.R.S. Received June 3, 1915.)

[Plate 7.]

The comprehensive investigations of Campbell* have shown that the spectra of the Wolf-Rayet stars contain in addition to lines due to hydrogen and helium, a number of lines which have not been identified with any spectrum which has hitherto been produced in the laboratory. Owing to the very diffuse character of the lines in the spectra of the Wolf-Rayet stars, accurate measurements of wave-length are impossible, and any identification of the lines with a terrestrial spectrum must, therefore, depend on the apparent coincidence of a relatively large number of lines with the spectrum produced in the laboratory.

* 'Astronomy and Astrophysics,' vol. 13, p. 448 (1894).

M. Wolf* has materially added to our knowledge in the more refrangible regions of the spectra of these stars, but in the less refrangible regions, few lines have been added to the list given by Campbell. In a recent investigation, Wright† has discussed the possible relations between the Wolf-Rayet stars and the planetary nebulæ.

In the spectra of these stars, the lines of the ζ Puppis series are very prominent, and from the high order of energy which is necessary to produce this series in the laboratory it might be expected that other lines in the Wolf-Rayet spectrum would be found in the enhanced lines of some terrestrial element.

Nicholson,‡ in his remarkable theoretical investigations of the spectra of the nebulæ and the Wolf-Rayet stars, has concluded that the Wolf-Rayet spectrum is due to evolution products of the more simple atomic systems which are responsible for the nebular lines, and his suggested arrangement of the lines in series, somewhat resembling those ordinarily found, would appear to strengthen the possibility of producing the Wolf-Rayet spectrum in the laboratory.

The writer has recently observed a spectrum, apparently associated with carbon, of which the principal lines would appear to coincide with some of the most conspicuous lines in the Wolf-Rayet spectrum.

The spectrum was produced by passing heavy condensed discharges through vacuum tubes containing hydrogen at a moderately low pressure, and which were provided with graphite or carbon electrodes. The electrodes consisted either of pencil leads, which had been heated to a white heat and subsequently treated with boiling nitric and hydrochloric acids, or thin rods cut from a specially pure block of carbon and treated in the same way. The electrodes were attached to platinum wires, which were sealed into the vacuum tubes in the usual way. The tubes were exhausted by means of a Gaede mercury pump and a heavy discharge was passed during the process of exhaustion. Pure hydrogen was admitted by heating, in a Bunsen flame, a small palladium tube connected with the vacuum tube. The gas thus admitted was pumped out, and this operation was repeated several times in order to wash out completely any trace of other gases from the tube.

The vacuum tubes, when freshly prepared, showed only the spectrum of hydrogen with a trace of the Ångström carbon oxide bands, but after running the tube for some time the hydrogen spectrum disappeared and nothing remained but a very brilliant spectrum showing the Ångström bands. At the

* 'Sitz. Heidelberger Akad. Wiss.,' Abh. 14 and 22 (1913).
† 'Astrophys. Journ.,' vol. 2, p. 466 (1914).
‡ See 'Monthly Notices, R.A.S.,' vol. 75, 4, p. 340.

same time, carbon was deposited on the walls of the capillary; it was thus necessary to use end-on tubes for the spectroscopic observations. When excited by a condensed discharge, with a spark-gap in the circuit, the tubes showed the line spectrum of carbon and in the more refrangible region the multitude of lines, due to oxygen and the glass walls of the capillary, which appear in every low pressure vacuum tube excited in this way. The most characteristic lines, however, were a group in the yellow-green. In Table I are given the principal lines in this spectrum.

Table I.

λ.	Intensity.	Remarks.
6583	8	Characteristic carbon pair.
6578	9	
5826·7	3	
5812·0	5	
5801·4	7	
5696·0	10	5694·1 Ångström and Thalén.
5592·1	8	
4651·6	8	
4650·4	8	
4647·6	10	
4267	10	Very strong carbon line.

With the exception of the very characteristic carbon lines at λλ6583, 6578, and 4267, the lines included in this list are only those lines which are enhanced by very powerful discharges. Other lines recorded* as carbon lines were present, and also a pair at λ5893 and a line at λ6098, which, Prof. Fowler informs me, have long been regarded as unrecorded carbon lines in the South Kensington laboratories.

Lockyer, Baxandall, and Butler,† with similar conditions of powerful electric discharge, have observed the pair λλ4650·8 and 4647·6 in vacuum tubes containing compounds of carbon, and have attributed these lines to carbon; they have also drawn attention to the coincidence of this pair with lines in the spectrum of ε Orionis, and have suggested that they may possibly also account for the line λ4652 of the Wolf-Rayet stars. These lines are undoubtedly identical with two lines given in the list.

A line has been recorded at λ5694·1 in the spectrum of carbon by Ångström and Thalén, but this line was not observed by Eder and Valenta or Gramont.‡ Thalén has recorded lines in the spectrum of aluminium at

* Kayser's 'Handbuch der Spektroscopie.'
† 'Roy. Soc. Proc.,' A, vol. 82, p. 532 (1909).
‡ Kayser, *loc. cit.*, vol. 5, p. 225.

$\lambda\lambda$ 5695·5 and 5592·5, but it is extremely unlikely that these are identical with the lines observed in the vacuum tubes. Aluminium lines are a common impurity in the spectra of vacuum tubes excited by powerful discharges, the pair at $\lambda\lambda$ 3961·7 and 3944·2 being generally present. This may be due to the aluminium electrodes which are usually employed, or to the alumina (usually about 4 per cent.) contained in the glass. The lines at $\lambda\lambda$ 5696 and 5592 could not be obtained from vacuum tubes provided with aluminium electrodes and filled in the manner described. Moreover, in the spectrum from the tubes with carbon electrodes, these lines were not accompanied by the line λ 5722 of intensity 10, which is given in Thalén's list of aluminium lines. The new lines are diffuse in character, and are therefore difficult to measure.

It is dangerous to assume the origin of any lines obtained from vacuum tubes under these conditions of electric discharge, but it would appear justifiable provisionally to assign the lines observed to carbon, since it has not been found possible to obtain them in the absence of carbon.

The new lines are best developed when the walls of the capillary are well coated with the carbon deposit, and are strongly enhanced by powerful discharges, relative to the ordinary carbon lines. This is especially true of the group at $\lambda\lambda$ 5827, 5812, 5801, which are scarcely visible with a weak condensed discharge. One may perhaps imagine a condition of still more powerful excitation, in which the spectrum of carbon would consist of the new lines, with faint lines at $\lambda\lambda$ 6583, 6578, and 4267 as the sole surviving representatives of the ordinary carbon spark spectrum, since these are its most characteristic lines.

We may now compare these lines with the spectrum of the Wolf-Rayet stars. In Table II is given a list of Wolf-Rayet lines. It consists essentially of Campbell's (*loc. cit.*) list, with the following modifications:—

(i) All lines due to hydrogen or helium have been omitted.

(ii) Two lines observed by Merrill* have been included.

(iii) Wright (*loc. cit.*) has pointed out that the band λ 5813 appears to vary in position in different stars, and in the star B.D. + 30°3639 can be seen to be composite, having components at $\lambda\lambda$ 5801, 5812, 5828. These wave-lengths have been substituted for the line λ 5813 in Campbell's list.

In Column I are given the Wolf-Rayet lines, and in Column II, under "vacuum tube," lines in the spectrum provisionally assigned to carbon.

For the line given by Campbell at λ 4273, Wolf (*loc. cit.*) finds in the stars 30°3639, 36°3956, and 35°4013 respectively the values $\lambda\lambda$ 4268·1, 4270, and 4269, a result which would indicate the possibility of this line being identical

* 'Lick Observatory Bulletin,' vol. 7, p. 129 (1913).

Table II.

I. Wolf-Rayet. λ.		II. Vacuum tube.
6583		6583, 6578
6548		
5848		
5828	5813 Campbell. Very strong	5826 ·7
5812		5812 ·0
5801		5801 ·4
5693	Very strong	5696 ·0
5593	Strong	5592 ·1
5472	Strong	
5284		
5250		
5131		
4940		
4787		
4652	Very strong	4651 ·6, 4650 ·4, 4647 ·6
4636	Strong	
4626	Strong	

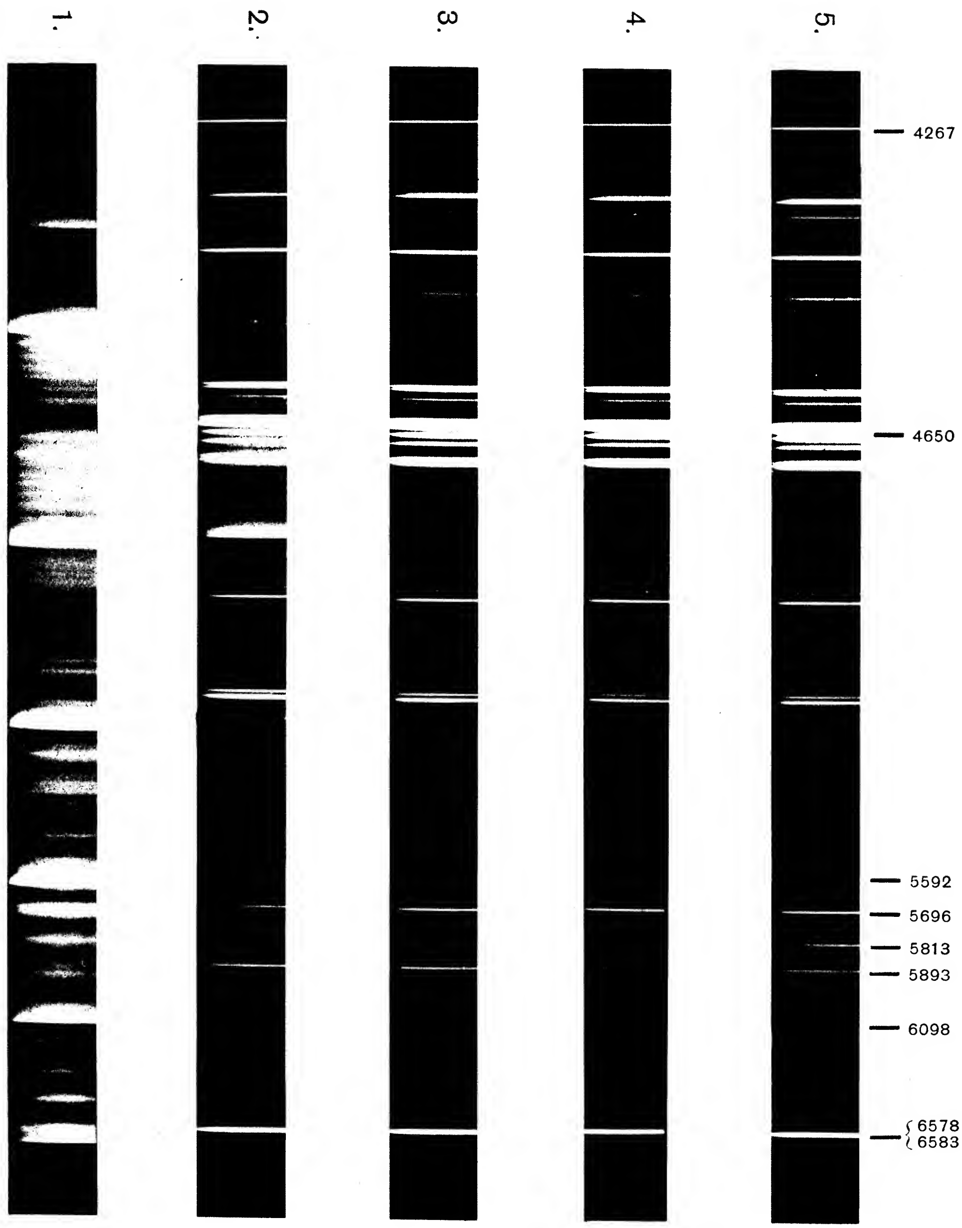
I. Wolf-Rayet. λ.		II. Vacuum tube.
4615		
4596		
4555		
4517	Strong	
4509	Very strong	
4504	Strong	
4493		
4480		
4466	Strong	
4457		
4442	Strong	
4416		
4369		
4334		
4318		
4273		4267
4260		
4228		
4063	Strong	

with the carbon line λ 4267. Campbell's (*loc. cit.*) results would appear to suggest a common origin for the lines λλ 5813, 5693, 5593, 4650, which in almost every case occur together. On the other hand, the visual intensity curves of the spectra of different stars* show that the relative intensities of these lines vary considerably in different stars. Similar variations of intensity can easily be produced in the lines observed in the vacuum tubes, the triplet at λ 4650 being produced with comparatively weak condensed discharges. The line λ 5696 is brought out with more powerful discharges, whilst the group λλ 5827, 5812, 5801, is strongly developed only by the most intense discharges. It will thus be seen that a considerable proportion of the stronger Wolf-Rayet lines are apparently coincident with lines in the vacuum tube spectrum. It cannot be claimed that the identity of the spectra has been fully established, but the results would appear to warrant the suggestion that the Wolf-Rayet lines in question may possibly be due to the same origin as the spectrum which has been described, and which is probably associated with carbon.

I should like to express my best thanks to Prof. Fowler for the valuable advice which he has given me.

In the plate, Nos. 1, 2, 3, 4, and 5 represent a series of the spectra obtained from a vacuum tube with successively increasing intensity of electric

* Campbell, *loc. cit.*, p. 7.


1.
2.
3.
4.
5.
4267
4650
5592
5696
5813
5893
6098
6578
6583

discharge. Thus No. 1 was obtained with an uncondensed discharge and No. 5 with a heavy condensed discharge. No. 2, with a moderate condensed discharge, shows lines of the ordinary carbon spark spectrum with $\lambda\lambda$ 4650 and 5696. In No. 3, the λ 5813 group and λ 5593 are just visible, whilst No. 5 shows these lines with considerable intensity; they are situated in a part of the spectrum to which the plates used are comparatively insensitive, and their photographic intensities are, in consequence, very small in comparison with their appearance in visual observations. In the more refrangible region, which in the plate was necessarily over exposed, strong lines due to oxygen, etc., from the walls of the tube are visible. The use of a filter to counteract the sensibility curve of the plate was only partly successful.

Hydrodynamical Problems Suggested by Pitot's Tubes.

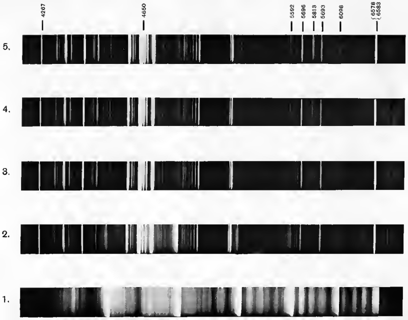
By Lord Rayleigh, O.M., F.R.S.

(Received June 5, 1915.)

The general use of Pitot's tubes for measuring the velocity of streams suggests hydrodynamical problems. It can hardly be said that these are of practical importance, since the action to be observed depends simply upon Bernoulli's law. In the interior of a long tube of any section, closed at the further end and facing the stream, the pressure must be that due to the velocity (v) of the stream, *i.e.* $\frac{1}{2}\rho v^2$, ρ being the density. At least, this must be the case if viscosity can be neglected. I am not aware that the influence of viscosity here has been detected, and it does not seem likely that it can be sensible under ordinary conditions. It would enter in the combination ν/vl, where ν is the kinematic viscosity and l represents the linear dimension of the tube. Experiments directed to show it would therefore be made with small tubes and low velocities.

In practice a tube of circular section is employed. But, even when viscosity is ignored, the problem of determining the motion in the neighbourhood of a circular tube is beyond our powers. In what follows, not only is the fluid supposed frictionless, but the circular tube is replaced by its two-dimensional analogue, *i.e.* the channel between parallel plane walls. Under this head two problems naturally present themselves.

The first problem proposed for consideration may be defined to be the flow


4267
4650
5592
5696
5813
5693
6098
6578
6583
5.
4.
3.
2.
1.